DENSITY OF HEDGEHOGS JUST AFTER A QUENCH

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Density of vortices and monopoles after a rapid pressure quench is predicted. Critical exponents are worked out in overdamped and underdamped limits.

I. INTRODUCTION.

Topological defects play a prominent role in many condensed matter systems, see e.g. [1] for a review. It was also suggested that they were an important ingredient of the early universe [2]. Topological defects can be generated in large numbers during a second order phase transition. The dynamics of such a transition to a symmetry broken phase has been an object of much recent attention because of its importance in the cosmological context [3] and in condensed matter physics [4]. An early estimate of defect density after a quench was given by Kibble [5]. It this theory, the speed of light is a dominant factor which determines the size of correlated domains. A more detailed scenario was put forward by Zurek [6], who emphasized the importance of the nonequilibrium dynamics of the order parameter. Recent experiments point to the latter theory.

In a recent paper [7] we calculated density of kinks in an underdamped one dimensional ϕ^4 theory. In this paper we give a comprehensive treatment of hedgehogs in $D = 2, 3$ spatial dimensions. A hedgehog is a point like defect which is topologically stable thanks to its asymptote at infinity. The asymptote is a topologically nontrivial map $S^D \to S^D$. A hedgehog should be distinguished from a texture the asymptote of which is a constant. The paper has essentially two parts. In the first we give a general outline of the formalism. In the second part the formalism is applied to vortices and monopoles.

II. MODEL.

Let us consider an $O(D)$ symmetric time dependent Ginzburg-Landau model in D spatial dimensions

$$\ddot{\phi}^\alpha(t, \vec{x}) + \gamma\dot{\phi}^\alpha(t, \vec{x}) - \frac{\partial}{\partial x^k}\frac{\partial}{\partial x^k}\phi^\alpha(t, \vec{x}) =$$
$$-2a(t)\phi^\alpha(t, \vec{x}) - 2[\phi^\beta(t, \vec{x})\phi^\beta(t, \vec{x})]\phi^\alpha(t, \vec{x}) + \eta^\alpha(t, \vec{x}) \ , \ (1)$$

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where $k = 1, \ldots, D$ are spatial indices and $\alpha, \beta = 1, \ldots, D$ are internal indices. Repeated indices imply summation. $\phi^\alpha(t, \vec{x})$'s are real. $\eta^\alpha(t, x)$'s are Gaussian white noises of temperature T with correlations

$$< \eta^\alpha(t, \vec{x}) >= 0 \ ,$$
$$< \eta^{\alpha_1}(t_1, \vec{x}_1)\eta^{\alpha_2}(t_2, \vec{x}_2) >=$$
$$2\gamma T \ \delta_{\alpha_1\alpha_2} \ \delta(t_1 - t_2) \ \delta^{(D)}(\vec{x}_1 - \vec{x}_2) \ . \quad (2)$$

The coefficient $a(t)$ is time dependent. We consider a linear quench

$$a(t) = \begin{cases} A & , \text{ if } t < 0 \\ A(1 - \frac{t}{\tau}) & , \text{ if } 0 < t < \tau\frac{A+1}{A} \\ -1 & , \text{ if } \tau\frac{A+1}{A} < t \end{cases} \quad (3)$$

Before the quench, for $t < 0$, the system is in a symmetric phase $(A > 0)$, during the quench, at $t = \tau$, it undergoes a transition from the symmetric phase $a(t < \tau) > 0$ to a broken symmetry phase $a(t > \tau) < 0$. Finally it settles down at $a(t) = -1$.

III. SOLITON TRANSFORM.

The model (1) in its symmetry broken phase $(a(t) = -1)$ admits pointlike topological defects or solitons

$$\phi^\alpha(\vec{x}) = S^\alpha(\vec{x} - \vec{z}). \quad (4)$$

$\vec{z}$ is an arbitrary location of the soliton. The soliton solution has D zero modes

$$Z_k^\alpha(\vec{x} - \vec{z}) = -\frac{\partial}{\partial z^k}S^\alpha(\vec{x} - \vec{z}) = \frac{\partial}{\partial x^k}S^\alpha(\vec{x} - \vec{z}) \ . \quad (5)$$

When field equations are linearized around the soliton solution (4), one obtains a discrete/continous spectrum of eigenmodes of the fluctuation operator. There are D zero modes (5) and a spectrum of modes with positive eigenvalues and eigenfunctions $P_\rho^\alpha(\vec{x} - \vec{z})$ labelled by a discrete/continuous index ρ. A field $\phi^\alpha(t, \vec{x})$ can be expanded as a sum of the soliton solution (4) with an appropriate choice of the soliton position $\vec{z}$ and a combination of positive eigenvalue eigenmodes with coefficients Φ_ρ,

$$\phi^\alpha(t, \vec{x}) = S^\alpha[\vec{x} - \vec{z}(t)] + \sum_{\rho \neq 0} \Phi_\rho(t)P_\rho^\alpha[\vec{x} - \vec{z}(t)] \ . \quad (6)$$

$\rho \neq 0$ in the sum stresses that the zero modes are excluded from the expansion, they are absorbed into the soliton location $\vec{z}(t)$. Given the order parameter $\phi^\alpha(t, \vec{x})$,

one can trace the soliton position $\vec{z}(t)$ with a help of a "soliton transform" of the order parameter

$$I^k(t, \vec{x}) = \int d^D y \; Z_k^\alpha(\vec{y}) \; \phi^\alpha(t, \vec{x} + \vec{y}) \; . \qquad (7)$$

$I^k[t, z\vec{(t)}] = 0$ for the field (6) because the zero modes (5) are orthogonal to the soliton solution (4) and to the eigenmodes $P_\rho^\alpha(\vec{x} - \vec{z})$.

The soliton position $\vec{z}(t)$ is a zero of the soliton transform, $I^k[t, z\vec{(t)}] = 0$, [8].

It may happen that there are several solitons in the system. In that case an expansion like (6) is possible around any soliton position $\vec{z}_1(t), \vec{z}_2(t), \ldots$. Each $\vec{z}(t)$ is a zero of the soliton transform. To count the solitons one should count the zeros of the soliton transform. An average density of solitons can be found as

$$n(t) = \int_{L^D} \frac{d^D x}{L^D} \; < \delta^{(D)}[\vec{I}(t, \vec{x})] \mid \frac{\partial(I^1, \ldots, I^D)}{\partial(x^1, \ldots, x^D)}(t, \vec{x}) \mid \; > \; ,$$

$$(8)$$

where the integration goes over a D-dimentional box of size L. Following similar steps as in [9], we obtain

$$n(t) = C_D \Big(\frac{-W''(t, r = 0)}{W(t, r = 0)} \Big)^{\frac{D}{2}} \; ,$$

$$W(t, |\vec{x}|) = < I^k(t, 0) I^k(t, \vec{x}) > \; ,$$

$$C_D = \begin{cases} \frac{1}{\pi} & , \text{ if } D = 1 \\ \frac{1}{2\pi} & , \text{ if } D = 2 \\ \frac{1}{\pi^2} & , \text{ if } D = 3 \end{cases} \qquad (9)$$

Once the correlation function $W(t, r)$ is known one can work out the average density of solitons in the system. In the considered cases the soliton transform does not distinguish solitons from antisolitons so Eq.(9) gives a total density of topological defects.

It is convenient to express $W(t, r)$ by a correlation function of the order parameter. The order parameter can be expanded in Fourier modes

$$\phi^\alpha(t, \vec{x}) = \int d^D p \; e^{i\vec{p}\vec{x}} \tilde{\phi}^\alpha(t, \vec{p}) \; . \qquad (10)$$

Thanks to translational invariance and the $O(D)$ symmetry of the model (1), a correlation function of the Fourier modes has the form

$$< \tilde{\phi}^{\alpha_1}(t, \vec{p_1}) \tilde{\phi}^{\alpha_2}(t, \vec{p_2}) > \equiv \frac{T \delta^{\alpha_1 \alpha_2} \delta^{(D)}(\vec{p_1} - \vec{p_2}) G(t, p_1)}{(2\pi)^D}.$$

$$(11)$$

This definition and the Fourier transform of the zero mode,

$$\tilde{Z}_k^\alpha(\vec{p}) = \int d^D x \; e^{-i\vec{p}\vec{x}} Z_k^\alpha(\vec{x}) \; ,$$

$$U(|\vec{p}|) = \tilde{Z}_k^\alpha(+\vec{p}) \tilde{Z}_k^\alpha(-\vec{p}) \; , \qquad (12)$$

leads to the following expression for the density of solitons

$$n(t) = C_D \Big(\frac{\int d^D p \; p^2 \; U(p) \; G(t, p)}{\int d^D p \; U(p) \; G(t, p)} \Big)^{\frac{D}{2}} \; . \qquad (13)$$

To find the density of solitons just after the quench (3) we need a correlation function $G(\tau, p)$ when the parameter $a(t)$ passes through the critical point and enters the symmetry broken phase. This correlation function is a topic of the next section.

IV. CORRELATION FUNCTION AT THE CRITICAL POINT.

As long as the system is still in the symmetric phase $(t < \tau)$, the order parameter ϕ^α can be regarded as a small fluctuation around its symmetric ground state $< \phi^\alpha(t, \vec{x}) >= 0$. It is justified to neglect the cubic term on the RHS of Eq.(1). A Fourier transform of the linearized Eq.(1) is

$$\ddot{\tilde{\phi}}^\alpha(t, \vec{p}) + \gamma \dot{\tilde{\phi}}^\alpha(t, \vec{p}) = -[2a(t) + p^2] \tilde{\phi}^\alpha(t, \vec{p}) + \tilde{\eta}^\alpha(t, \vec{p}) \; , \quad (14)$$

$$< \tilde{\eta}^\alpha(t, \vec{p}) >= 0 \; ,$$

$$< \tilde{\eta}^{\alpha_1}(t_1, \vec{p_1}) \tilde{\eta}^{\alpha_2}(t_2, \vec{p_2}) >=$$

$$\frac{2\gamma T}{(2\pi)^D} \; \delta^{\alpha_1 \alpha_2} \; \delta(t_1 - t_2) \; \delta^{(D)}(\vec{p_1} - \vec{p_2}) \; .$$

$$(15)$$

The correlation function $G(\tau, p)$ can be easily worked out in two opposite limits.

A. Underdamped limit, $\gamma \to 0$.

The underdamped limit is achieved if $2a(t) >> \gamma^2/4$ from the beginning of the quench to the freeze-in time $\hat{t}$. In this limit the system (14) can be mapped to the equivalent model

$$\dot{\tilde{\phi}}^\alpha(t, \vec{p}) = -\frac{\gamma}{2} \tilde{\phi}^\alpha(t, \vec{p}) + n^\alpha(t, \vec{p}) \; ,$$

$$< n^\alpha(t, \vec{p}) >= 0 \; ,$$

$$< n^{\alpha_1}(t_1, \vec{p_1}) n^{\alpha_2}(t_2, \vec{p_2}) >=$$

$$\frac{2\gamma T \; \delta^{\alpha_1 \alpha_2} \; \delta(t_1 - t_2) \; \delta^{(D)}(\vec{p_1} - \vec{p_2})}{(2\pi)^D [2a(t_1) + p_1^2]} \; . \qquad (16)$$

To see the nature of this equivalence let us consider three examples. If $a(t) = const$, then both systems have the same correlation

$$G(t, p) = \frac{1}{2a + p^2} \; . \qquad (17)$$

Their response to the change of parameters is also the same. In particular they have the same relaxation time.

For example, if $a(t) = const$ and the temperature jumps from $T = 0$ for $t < 0$ to $T > 0$ for $t > 0$, then the correlation function for $t > 0$ is

$$G(t,p) = \frac{1}{2a + p^2}[1 - e^{-\gamma t}] \ ,$$

$$G(t,p) = \frac{1}{2a + p^2}[1 - e^{-\gamma t} + e^{-\gamma t}O(\frac{\gamma}{\sqrt{2a + p^2}})] \quad (18)$$

for the models (16) and (14) respectively. $O(\ldots)$ are oscillating terms which are negligible for small γ. In this limit the response of the two systems is the same. Another example is a response to a sudden change in the parameter a from $a = a_-$ for $t < 0$ to $a = a_+$ for $t > 0$

$$G(t,p) = \frac{1 - e^{-\gamma t}}{2a_+ + p^2} + \frac{e^{-\gamma t}}{2a_- + p^2} \ ,$$

$$G(t,p) = \frac{1 - e^{-\gamma t}}{2a_+ + p^2} + \frac{e^{-\gamma t}}{2a_- + p^2} +$$
$$O(\frac{\gamma e^{-\gamma \tau}}{\sqrt{2a_+ + p^2}}, \frac{\gamma e^{-\gamma \tau}}{\sqrt{2a_- + p^2}}) \ , \quad (19)$$

where $O(\ldots)$ are once again oscillations negligible for $\gamma \to 0$. The two considered systems respond in the same way to the impulsive perturbations of their parameters. The impulsive perturbations contain all frequencies so the models can be considered equivalent in the underdamped limit.

In the underdamped limit the relaxation times of all the Fourier modes are the same and equal to $1/\gamma$. One can talk about a relaxation time of the system as a whole. As $a(t)$ begins to change at $t = 0$ the system tries to adapt to the changes. It can not react faster than on the time scale of its relaxation time $1/\gamma$. The correlations are effectively frozen at the freeze-in time $\hat{t}$, when the time left to the transition is equal to the relaxation time, $(\tau - \hat{t}) = \frac{1}{\gamma}$. At this instant $a(\hat{t}) = \frac{A}{\gamma \tau}$, so the frozen correlation function is

$$\hat{G}_{\gamma \to 0}(\tau,p) = \frac{1}{\frac{2A}{\gamma \tau} + p^2} \ . \quad (20)$$

The frozen correlation length can be identified as

$$\hat{\xi}_{\gamma \to 0} = \sqrt{\frac{\gamma \tau}{2A}} \sim \tau^{1/2} \ . \quad (21)$$

These asymptotic results are valid for $\tau >> 1/\gamma$. The system remains in the underdamped limit until the freeze-in time, $2a(t < \hat{t}) >> \gamma^2/4$, if the condition

$$8A > \tau \gamma^3 \quad (22)$$

is satisfied. If $\tau << 1/\gamma$, then the system is frozen from the beginning of the quench and the frozen correlation length is $\hat{\xi} = 1/\sqrt{2A}$.

The analysis so far was on a heuristic level. The equation (16) is of first order in time, so the exact correlation function of the system (16) can be easily worked out. The formal solution of the model (16) is

$$\tilde{\phi}^\alpha(\tau,\vec{p}) = \int_{-\infty}^{\tau} dt_1 \ e^{-\gamma(\tau - t_1)/2} n^\alpha(t_1, p). \quad (23)$$

With this solution the correlation function is

$$G(\tau,p) = \frac{e^{-\gamma \tau}}{2A + p^2} + \int_0^{\gamma \tau} dx \ \frac{e^{-x}}{\frac{2A}{\gamma \tau}x + p^2} \ . \quad (24)$$

It turns out that Eq.(20) is indeed the asymptote of the exact solution for $\tau >> 1/\gamma$.

B. Overdamped limit, $\gamma \to \infty$.

In the overdamped limit $(\gamma^2/4 >> 2a(t))$ the second order time derivative on the LHS of Eqs.(1,14) can be neglected. Eq.(14) is simplified to

$$\gamma \dot{\tilde{\phi}}^\alpha(t,\vec{p}) = -[2a(t) + p^2]\tilde{\phi}^\alpha(t,\vec{p}) + \tilde{\eta}^\alpha(t,\vec{p}) \ . \quad (25)$$

In an overdamped system a relaxation time of a given Fourier mode depends on its momentum p and equals $\gamma/2[2a(t) + p^2]$. There is no universal relaxation time of the system as a whole. Short wave lengths freeze in later than the long wave lengths. For a given momentum p, the relaxation time is equal to the time left to the transition, if $(\tau - \hat{t}_p) = \gamma/2[2a(\hat{t}_p) + p^2]$. On this heuristic level the long wave length limit of the frozen correlation function is

$$\hat{G}_{heuristic}(\tau,p) \approx \frac{1}{\sqrt{\frac{\gamma A}{\tau} + p^2}} \ . \quad (26)$$

The frozen correlation length can be identified as

$$\hat{\xi}_{heuristic} = (\frac{\tau}{\gamma A})^{\frac{1}{4}} \sim \tau^{1/4} \ . \quad (27)$$

The model (25) has an exact solution

$$\tilde{\phi}^\alpha(\tau,\vec{p}) = \int_{-\infty}^{\tau} dt_1 \ \exp\{-\int_{t_1}^{\tau} dt_2 \ [2a(t_2) + p^2]/\gamma\} \ \tilde{\eta}(t_1,\vec{p}) \ . \quad (28)$$

This solution, the correlations in Eq.(14) and the explicit form of $a(t)$, see Eq.(3), give an exact correlation function at $t = \tau$

$$G(\tau,p) = \frac{e^{-2p^2\tau/\gamma}}{2A + p^2} +$$
$$\sqrt{\frac{\pi \tau}{2\gamma A}}e^{\frac{\tau p^4}{2\gamma A}}[Erf(\sqrt{\frac{\tau}{2\gamma A}} (2A + p^2)) - Erf(\sqrt{\frac{\tau}{2\gamma A}} (p^2))] \ . \quad (29)$$

If τ is much longer than the relaxation time of the $p = 0$ mode before the quench, $\tau >> 2\gamma A$, the correlation function takes an asymptotic form

$$G(\tau, p) \approx \sqrt{\frac{2\tau}{\gamma A}} e^{\frac{p^4 \tau}{2\gamma A}} \int_{\sqrt{\frac{\tau}{2\gamma A}} p^2}^{+\infty} dx \, e^{-x^2} \quad . \qquad (30)$$

The frozen correlation length can be identified as

$$\hat{\xi}_{\gamma \to \infty} = \left(\frac{\tau}{\pi \gamma A}\right)^{\frac{1}{4}} \quad . \qquad (31)$$

V. DENSITY OF VORTICES.

In two dimensions ($D = 2$) the model (1) admits a topologically stable vortex solution

$$S^\alpha(\vec{x}) = f(\rho)[\delta^{\alpha 1} \cos(\varphi) + \delta^{\alpha 2} \sin(\varphi)] \quad , \qquad (32)$$

where ρ, φ are polar coordinates and the profile function interpolates between $f(0) = 0$ and $f(\infty) = 1$. A single vortex has infinite energy, the IR divergence is due to the long distance asymptote

$$S^\alpha_\infty(\vec{x}) = [\delta^{\alpha 1} \cos(\varphi) + \delta^{\alpha 2} \sin(\varphi)] \quad . \qquad (33)$$

The asymptote is scale invariant. The asymptote determines the long wave length asymptote of the Fourier transform, which is

$$U(p) \sim \frac{1}{p^2} \qquad (34)$$

up to a numerical factor.

A. Underdamped limit.

Combination of the formulas (20,13,34) should give the density of vortices for large τ. The numerator integral in (13) is proportional to

$$\int_\Lambda^{+\infty} p \, dp \, p^2 \, U(p) \, G(\tau, p) \stackrel{\tau \to \infty}{\approx}$$

$$\int_\Lambda^{+\infty} p \, dp \, p^2 \, U(p) \frac{1}{\frac{2A}{\gamma \tau} + p^2} \stackrel{\tau \to \infty}{\approx}$$

$$\int_\Lambda^\infty p \, dp \, U(p) \stackrel{\Lambda \to 0}{\approx} - \ln(\Lambda) \quad . \qquad (35)$$

Λ is an IR regulator. The denominator integral is proportional to

$$\int_\Lambda^{+\infty} p \, dp \, U(p) \, G(\tau, p) \stackrel{\tau \to \infty}{\approx}$$

$$\int_\Lambda^{+\infty} p \, dp \, U(p) \frac{1}{\frac{2A}{\gamma \tau} + p^2} \stackrel{\Lambda \to 0}{\approx}$$

$$\frac{\gamma \tau}{2A} \int_\Lambda \frac{dp}{p} = -\frac{\gamma \tau}{2A} \ln(\Lambda) \quad . \qquad (36)$$

We manipulated with the order of the limits $\tau \to \infty$ and $\Lambda \to 0$ so as to get rid of the IR divergence and obtain a finite regularized asymptote

$$n_{REG.} \approx \frac{A}{\pi \gamma \tau} \quad . \qquad (37)$$

The results for vortices are dominated by the IR divergence, which is due to the asymptote (33). The asymptote is scale invariant so the density of vortices is just an inverse squared of the frozen correlation length (21).

B. Overdamped limit.

In this limit we combine the formulas (30,13,34). The numerator in Eq.(13) is proportional to

$$\int_\Lambda^{+\infty} p \, dp \, p^2 \, U(p) \, G(\tau, p) \stackrel{\tau \to \infty}{\approx}$$

$$\int_\Lambda^{+\infty} p \, dp \, U(p) \stackrel{\Lambda \to 0}{\approx} - \ln(\Lambda) \quad . \qquad (38)$$

The denominator is

$$\int_\Lambda^{+\infty} p \, dp \, U(p) \, G(\tau, p) \stackrel{\tau \to \infty}{\approx}$$

$$\sqrt{\frac{2\tau}{\gamma A}} \int_\Lambda^{+\infty} p \, dp \, U(p) \, e^{\frac{\tau p^4}{2\gamma A}} \int_{\sqrt{\frac{\tau}{2\gamma A}} p^2}^{+\infty} dx \, e^{-x^2} \stackrel{\Lambda \to 0}{\approx}$$

$$\sqrt{\frac{\pi \tau}{\gamma A}} \int_\Lambda p \, dp \, U(p) = -\sqrt{\frac{\pi \tau}{\gamma A}} \ln(\Lambda) \quad . \qquad (39)$$

The regularized asymptote of the density of vortices is

$$n_{REG.} \approx \frac{1}{2\pi^{3/2}} \sqrt{\frac{\gamma A}{\tau}} \quad . \qquad (40)$$

The $1/2$ critical exponent is consistent with the numerical results in [10].

VI. DENSITY OF MONOPOLES.

The analysis of monopoles in $D = 3$ dimensions is much the same as that of vortices in $D = 2$ dimensions. The monopole configuration is

$$S^\alpha(\vec{x}) = f(r)[\delta^{\alpha 1} \sin(\theta) \cos(\varphi) + \delta^{\alpha 2} \sin(\theta) \sin(\varphi) + \delta^{\alpha 3} \cos(\theta)] \quad , \qquad (41)$$

where r, θ, φ are spherical coordinates. The profile function interpolates between $f(0) = 0$ and $f(\infty) = 1$. A single monopole has infinite energy, the IR divergence is due to the long distance asymptote

$$S^a_\infty(\vec{x}) = [\delta^{a 1} \sin(\theta) \cos(\varphi) + \delta^{a 2} \sin(\theta) \sin(\varphi) + \delta^{a 3} \cos(\theta)] \quad . \qquad (42)$$

The asymptote is scale invariant. The asymptote determines the long distance behaviour of the Fourier transform, which is

$$U(p) \sim \frac{1}{p^4} \tag{43}$$

up to a numerical factor. Similarly as for vortices this singularity alone determines the critical exponents.

A. Underdamped limit.

Combination of the formulas (20,13,43) gives the density of monopoles for large τ. The numerator integral in (13) is proportional to

$$\int_{\Lambda}^{+\infty} p^2 \, dp \, p^2 \, U(p) \, G(\tau, p) \overset{\tau \to \infty}{\approx}$$

$$\int_{\Lambda}^{+\infty} p^2 \, dp \, p^2 \, U(p) \, \frac{1}{\frac{2A}{\gamma\tau} + p^2} \overset{\tau \to \infty}{\approx}$$

$$\int_{\Lambda}^{\infty} p^2 \, dp \, U(p) \overset{\Lambda \to 0}{\approx} \frac{1}{\Lambda} \quad . \tag{44}$$

Λ is an IR regulator which is provided either by the system size or by an average distance between monopoles and antimonopoles. The denominator integral is proportional to

$$\int_{\Lambda}^{+\infty} p^2 \, dp \, U(p) \, G(\tau, p) \overset{\tau \to \infty}{\approx}$$

$$\int_{\Lambda}^{+\infty} p^2 \, dp \, U(p) \, \frac{1}{\frac{2A}{\gamma\tau} + p^2} \overset{\Lambda \to 0}{\approx}$$

$$\frac{\gamma\tau}{2A} \int_{\Lambda} \frac{dp}{p^2} = \frac{\gamma\tau}{2A} \frac{1}{\Lambda} \quad . \tag{45}$$

We manipulated with the order of the limits $\tau \to \infty$ and $\Lambda \to 0$ so as to get rid of the IR divergence and obtain a finite regularized asymptote

$$n_{REG.} \approx \frac{1}{\pi^2} \left(\frac{2A}{\gamma\tau}\right)^{\frac{3}{2}} \quad . \tag{46}$$

The asymptote (33) is scale invariant; the density of monopoles scales like an inverse qubed of the frozen correlation length (21).

B. Overdamped limit.

In this limit we combine the formulas (30,13,43). The numerator integral in Eq.(13) is proportional to

$$\int_{\Lambda}^{+\infty} p^2 \, dp \, p^2 \, U(p) \, G(\tau, p) \overset{\tau \to \infty}{\approx}$$

$$\int_{\Lambda}^{+\infty} p^2 \, dp \, U(p) \overset{\Lambda \to 0}{\approx} \frac{1}{\Lambda} \quad . \tag{47}$$

The denominator is

$$\int_{\Lambda}^{+\infty} p^2 \, dp \, U(p) \, G(\tau, p) \overset{\tau \to \infty}{\approx}$$

$$\sqrt{\frac{2\tau}{\gamma A}} \int_{\Lambda}^{+\infty} p^2 \, dp \, U(p) \, e^{\frac{\tau p^4}{2\gamma A}} \int_{\sqrt{\frac{\tau}{2\gamma A}} p^2}^{+\infty} dx \, e^{-x^2} \overset{\Lambda \to 0}{\approx}$$

$$\sqrt{\frac{\pi\tau}{\gamma A}} \int_{\Lambda} p^2 \, dp \, U(p) = \sqrt{\frac{\pi\tau}{\gamma A}} \frac{1}{\Lambda} \quad . \tag{48}$$

The regularized $\tau >> 2\gamma A$ asymptote of the density of vortices is

$$n_{REG.} \approx \frac{1}{\pi^{11/4}} \left(\frac{\gamma A}{\tau}\right)^{\frac{3}{4}} \quad . \tag{49}$$

VII. CONCLUSION.

We considered density of hedgehogs just after a rapid pressure quench. For large enough quench time τ the density depends on τ like

$$n \sim \tau^{-\sigma} \quad . \tag{50}$$

Our findings are summarized in a table.

		σ	
D	DEFECT	$\gamma \to 0$	$\gamma \to \infty$
2	VORTEX	1	$\frac{1}{2}$
3	MONOPOLE	$\frac{3}{2}$	$\frac{3}{4}$

The critical exponent depends on the degree of damping. The underdamped critical exponent is twice the overdamped critical exponent for a given defect. It is not surprising as the two limits are dominated by a second order and a first order time derivative respectively.

Global vortices and global monopoles interact by long distance confining potentials. Their density just after a quench is merely an initial condition for their subsequent rapid recombination described by phase ordering kinetics [11]. Their critical exponents apply to local vortices/monopoles coupled to a gauge field provided that the gauge field equilibrates with the matter field rather slowly [10]. These local defects have finite energy and interact weakly so their subsequent recombination is purely diffusive.

Topological textures like Bloch waves for $D = 1$, skyrmions for $D = 2$, magnetic bubbles and Hopfions for $D = 3$ require a separate study [12].

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